Economics and/or Mathematics, Equilibrium and Aggregation in Economic Science

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Abstract

The dominating approach in economic science is the neoclassical theory. It is built on an axiomatic structure which creates an economic man who is rational and optimizing. The axioms also allow additive aggregation so the individuals can be aggregated into a sort of society in which complete efficiency is obtained with respect to the use of productive resources in relation to the welfare of the society given an initial resource distribution. This paper shows that a closer inspection of the axiomatic structure in relation to the so called Arrow's paradox leads to the conclusion that although the neoclassical optimization approach may well describe the individual behavior, but any form of aggregation to the economy as a whole is basically built on a misinterpretation of the axiomatic structure and thus on false logics.

Keywords: neoclassical theory, economic rationality, general equilibrium, aggregation.

1. Introduction

Economics is a social science although it sometimes appears as a sub-division of logical/mathematical sciences. Certainly many deep going understandings of the society has been produced by logical/mathematical investigations but basically we must observe the reality in all its, sometimes, contradictive appearances. The dream of finding the general model which gives an answer to the most difficult questions of the social reality is understandable but unfortunately so far, just a dream. In this paper we will mainly discuss aggregation and equilibrium. For those attacking economic theory and methodology there are many angles of attack for example the assumed crude psychology underlying *the economic man*, the lack of ethical considerations, the deterministic methodology, the simplifying assumptions of the social complex and so on. The problem of aggregation, which is built on the particular concept of general equilibrium used in the neoclassical theory, is a central methodological problem in economics. Interestingly it seems to the author that physics and economics are the two sciences with the most explicit debate on the problem of aggregation. In economics we have one approach, the neoclassical one, which claims to have a method of aggregation while another one, the heterodox, with Keynesians at the front, claims that economics can only deal with observable macroscopic variables to find viable patterns for the macroscopic economic behavior, and they deny the possibility of aggregating people into a society, at least when it concerns additive aggregation.

2. Aggregation

Aggregation can be defined in the simplest way as implementing some observed characteristics of individual persons/items to also characterize the group per se which these individual persons/items belong to according to some kind of grouping. The aggregation can be of different character. In physics we can take a simple example from home-chemistry. We can mix sugar with flour, egg, yeast and some fruits and then get a tasty cake after baking it in the oven, but if we only mix sugar, flour, egg, yeast and some fruits, not baking it in the oven, we will have a rather unappetizing hotchpotch. The former is a non-additive aggregation since the contained ingredients merge to a new consistent form and the latter an additive aggregation since the ingredients are mixed but basically keep their chemical forms. If we mix sugar and flour and put a match to it nothing happens, an additive aggregation, while if we mix sugar with a certain chloride and put a match to it we will have an explosion, indeed a non-additive aggregation. The organism which we call society is remarkable since it seems to be a conscious organization in some dimensions, depending upon some almost universal rules in other dimensions but still some dimensions seem to be best described by chaos modeling.

Amartya Sen makes a good point of the amoeba-like dynamics of the majority principle as a way of aggregate preferences: *When distributional issues dominate and when people seek to maximize their own "shares" without concern for others (as, for example the cake division problem, with each preferring any division that increases her own share, no matter what happens to the others), then the majority rule will tend to be thoroughly inconsistent. But when there is a matter of national outrage (for example, in response to the inability of a democratic government to prevent a famine), the electorate may be reasonably unequivocal and thoroughly consistent.*

(Sen 2002:76)

Scientifically we may try to solve the problem by limiting the dimensionality of the problem, as is done when we look upon "sciences" as economics, sociology anthropology and so on as non-communicating containers. This type of "solution" is however in vain. It builds on an even worse form of additive aggregation namely that changes of the dimensionality of a structure leave the original structure unaffected. Technically this way of reasoning is indeed deceitful since we in such a discussion enter the very logical character of the variables/facts we employ for the analysis, atomistic versus complexes. Economics is the social science which most explicitly addresses the problem of aggregation and we have an intra-science conflict which has lasted for some 130 years. Economics is much criticized for the transformation of real structures into mathematical ones and the poor, not to say non-existent, psychological and sociological analysis of the individual and the society. The most elaborated theory, *the neoclassical theory*, has indeed a formalized form of individual psychology which also is underpinning the aggregate behavior in the society in relation to economic exchange. The most viable alternative to this approach, Keynesianism (of different forms), avoids prescribing the individual decisions and stick to the aggregate behavior of the economy. John Maynard Keynes actually sets some form of code in treating individual behavior when he tells us:

The fundamental psychological law, upon which we are entitled to depend with great confidence both a priori from our knowledge of human nature and from detailed facts of experience, is that men are disposed, as a rule on the average, to increase their consumption as their income increase, but not as much as the increase in their income.

(Keynes 1973[1936]: 96).

As we see Keynes bases one of his most important concepts, *the marginal propensity to consume*, on very aggregate considerations and discuss only individuals on the average. This seems to be the same for most other concepts he uses. Thusthere is no attempt to describe individuals other than on the average for the analyzed society in question. Concerning Keynes with respect to his entire writings and his analytical approach the impression of quotes like the one above is wrong. May be he did not have any attempt to develop some psychological theory of the individual but his structural understanding of the economy had a philosophical debt still not anticipated. Furthermore the so called Keynesianism was and to a great extent still is more based on the interpretations by John Hicks, Paul Samuelson and other economists basically inspired by the neoclassical theory who wanted to achieve some form of synthesis between the neoclassical theory and Keynes' ideas. Anyway the Keynesian theory as it appears lacks any clear links to individual behavior per se, and implies that the economic analysis concerns the highest possible aggregate levels with few links to the structural questions on different aggregate and individual levels. This led to a fundamental shift in economic thinking in the end of the 1960s and the beginning of the 70s, when the ruling analytical approach, a rather mechanic form of Keynesianism, was unable to foresee, explain and prescribe policy with respect to the structural changes in international economic relations in the aftermath of the break-down of the Bretton Woods system. As a matter of fact one of the biggest newspapers in Sweden had an obituary notice, at the editorial page, of the death of the theories of John Maynard Keynes. The general feeling was that structural explanations were needed and the only available, of theoretical importance, was the neoclassical theory. It was necessary with some form of microeconomic underpinning of the macroeconomic behaviour of the economy and the neoclassical theory was believed to provide such an underpinning. This belief was strengthened when Robert Lucas and Sargent & Wallace developed the so called rational expectation approach based on a microeconomic approach (Muth: 1962). In the 1970s the discussions were almost entirely kept within and among universities but Lucas' ideas (Lucas: 1981[1966]) based on the so called Lucas critique (Lucas: 1976) raised a lot of ideological/political enthusiasm and the ideas reached the political level relatively fast so from the beginning of the 1980s we have got an increasing political influence of these ideas which reached some form of peak when Lucas received the Economic Price in Memory of Alfred Nobel 1997.

In his presidential address to the American Economic Association 2003 he claimed that the 'central problem of depression-prevention has been solved, for all practical purposes, and has in fact been solved for many decades'.

As mentioned above the Keynesianism brake-down in the beginning of the1970s because of the lack of structural analysis, we thus may interpret Lucas' claim that the neoclassical theory, enlarged with the rational expectation hypothesis, provides us not only with ordinary aggregate business cycle information but also with sufficient structural information to be able to exercise a correct and efficient economic policy. Since we know that Lucas' claim perhaps was a bit exaggerated looking at what actually has happened since 2003, even the most analyzed market, the financial, experienced a catastrophe which had deep structural effects on the financial market behavior and vast secondary effects on the real economy, it could be of value to look at economic theory with respect to theoretical approaches considering the problem of aggregation. To my understanding economics is one of the social sciences which most explicitly deals with the problems of aggregation so although solutions can be criticized and even if there is no common approach among economists the aggregation problem is still one of the most important methodological problems. We will base our analysis particularly on Ekstedt & Fusari (2010) and Ekstedt (2015) and will touch on some of the fundamental issues in analytical philosophy during the 20th century and on questions which are fundamental in natural sciences and burning in social sciences and have to be solved in order to achieve a scientific methodology which is basically common. We sometimes blame social sciences in general and economics in particular to try to copy the methodology of natural sciences.

In a way such blame is relevant when methodological allusions are naïve but still the natural sciences use logics and mathematics in a very conscious way where variables are defined to a high degree through repetitive experiments which refines the analysis, although sometimes the analysis runs into paradoxes showing that premises and variable definitions are exhausted. When it comes to social sciences however we still are in the stage where important groups of scientists question mathematical and logical methods *in toto*. We claim that the basic difference between natural and social sciences are to be found in the fact that natural sciences deals with objects, thus the scientist is outside the set of variables under study, while social sciences deals with subjects, where the scientist is always a part of the set of variables under study. This means that while natural sciences in most cases can avoid the problem of Gödel's paradox this is not the case in social sciences. However this difference is not about whether logical/mathematical methodology can/should/shouldn't be used but about choosing consistent frames and variable definitions in different studies. Fundamentally however is that humans are to be seen as subjects and thus, locally and temporally, final causes. That means that actions can be studied as well as oral testimonies but the former need interpretation to enter into a causal chain and the latter is based on interaction between researcher and the subject in question. Consequently we have to remind ourselves of the essence of David Hume's analysis of the concept of causality. As we easily see the question of aggregation is completely different between natural and social sciences. So let us set our minds to aggregation as a concept.

3. Aggregation as a Concept

The simplest form of aggregation is the additive; a set of individuals forms a group given a specific context. Those who are watching a football match at a specific arena are interesting just as an additive assembly from many aspects, security/fire aspects, and income aspects given the price of tickets, the amount of consumed hot dogs and so on. However if we allow for differences in the features between possible subgroups, for example hooligans, young men, young men with fiancées and families, things become more difficult since that will possibly affect the behavior of and between the subgroups relative each other and thus the behavior of the whole group. Thus in the latter case the additive aggregation will convey a rather incomplete information, which probably calls for somewhat deeper, sometimes even scientific, investigations. Just this simple example leads us to the conclusion that an aggregate composition might not only be the simple additive number of individuals but also is dependent upon the structure of subgroups among individuals. Consequently we understand that the aggregate based on addition will only in special cases tell us something about the behavior of the group as a whole. The next question is then about the links between the aggregate and the individuals and vice versa.

If we deal with an additive aggregate we will have reversibility of the aggregate process, thus the precise initial character of the individuals can be achieved by a simple decomposition and furthermore there we achieve the same analytical results if we start from an average of the individuals or from analysing each individual and then add the results or if we analyse the average as a whole. This kind of things appears in chemistry when we have a simple mixture but we also have compounds where the structure of the included individual parts as we gave an example above from the kitchen chemistry when baking a cake.

When our task is to analyze social structures, behavior and conflicts we can hardly assume that humans are atoms according to the ancient Greek tradition but display a more complex nature, which we can summarize in the concept *subject* meaning as said above that a subject is a local and temporal final cause. From a logical and mathematical point of view such a change in the conceptual meaning of a variable is indeed striking with respect to its analytical consequences. So basically we may say that aggregation to a great extent also is linked to the question of reversibility and irreversibility and is also linked to the subject/object problem. Thus when it comes to aggregation we may say that atomistic variables are almost always possible to aggregate additively while complexes, which subjects must be seen as, are not. This holds almost generally for mathematics/probability theory/statistics, and also probably holds for general logical analysis although it is a somewhat heroic generalization. Thus a logical/mathematical analysis with respect to subjects must be limited within very precise frames which make us able to transform the very aspects to be treated as if they were atomistic variables. This problem is precisely the problem of generalizing analytical results as for example in any kind of statistics based on the strong/weak law on large numbers and this problem is fundamental. When we look at natural sciences this problem is equally important and demanding but since these sciences often work with more inert structures they can design experiments in order to refine the specifications of used variables and thus they might be able to reach models with relatively consistent and precise parameter structures. As a matter of fact this is the basic content of a famous letter from John Maynard Keynes to Roy Harrod 10th of July 1938 (Keynes: Collected Works):

My point against Tinbergen is a different one. In chemistry and physics and other natural sciences the object of experiment is to fill in the actual values of the various quantities and factors appearing in an equation or a formula; and the work when done is once and for all. In economics that is not the case, and to convert a model into a quantitative formula is to destroy its usefulness as an instrument of thought. Tinbergen endeavours to work out the variable quantities in a particular case, or perhaps in the average of several particular cases, and he then suggest that the quantitative formula so obtained has general validity. Yet in fact, by filling in figures, which one can be quite sure will not apply next time, so far from increasing the value of his instrument, he has destroyed it. All the statisticians tend that way. Colin, for example, has recently persuaded himself that the propensity to consume in terms of money is constant at all phases of the credit cycle. He works out a figure for it and proposes to predict by using the result, regardless of the fact that his own investigations clearly show that it is not constant, in addition to the strong a priori reasons for regarding it as most unlikely that it can be so.

We remind of the fact that Tinbergen was one of the first to receive the *Economic Price in the Memory of Alfred Nobel* and basically it was for his econometric work. However Keynes' attitude is quite interesting since he addresses mathematical modelling in a different way to what is usual in economics. To him making an economic model is to study/test the consistency of a certain type of mathematical forms of the economic reasoning like engineers building models of a bridge and try to test the specific characteristics of strength, adaptability and flexibility of different forms. All such tests are of limited value since the models are not of full size and built at the very place, which would be a somewhat expensive way of testing strength, but at least some comparative studies of forms could be performed and be of value within some limits. The great scientist in biology D'Arcy Thompson in the beginning of the 20th century made a very precise comment about nature but this comment could also be applied to modelling when using the common expression *ceteris paribus,*it is not wrong per se but we need to be aware of the fact that the processes we describe in the model are of limited dimensionality and when it is implemented in the crude reality the environment is magnified as well as of an drastically increased dimensionality. D'Arcy Thompson describes when the substance of an entity grows out of its form (D'Arcy Thompson 1992:16-7):

We call a thing big or little with reference to what it is wont to be, as when we speak of a small elephant or a large rat; and we are apt accordingly to suppose that size makes no other or more essential difference, and that Lilliput and Brobdingnag are all alike, according as we look at them through one end of the glass or the other. All this is true of number, and of relative magnitude. The Universe has its endless gamut of great and small, of near and far, of many and few. Nevertheless, in physical science the scale of absolute magnitude becomes a very real and important thing; and a new and deeper interest arises out of the changing ratio of dimensions when we come to consider the inevitable changes of physical relations with which it is bound up. The effect of scale depends not on a thing in itself, but in relation to its whole environment or milieu; it is in conformity with the thing's 'place in Nature', its field of action and reaction in the Universe. Everywhere Nature works true to scale, and everything has its proper size accordingly.

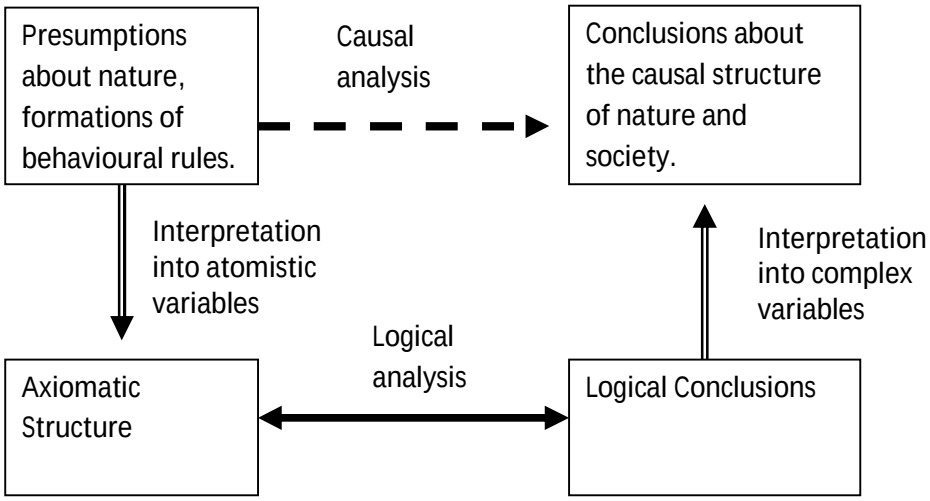
(Original emphasizes)

The quote from D'Arcy Thompson cannot be enough emphasized in our analysis based on subsets of the reality. As a matter of fact Wittgenstein also touches on this Tractatus proposition 6.211 (1921[1974]):

Indeed in real life a mathematical proposition is never what we want. Rather, we make use of mathematical propositions only in inferences from propositions that do not belong to mathematics to others that likewise do not belong to mathematics. (In philosophy the question, "What do we actually use this word or this proposition for?" repeatedly leads to valuable insights).

Thus our scientific job can be described as an analysis of subspaces of reality and then a transformation of believed important concepts into logical/mathematical concepts; then we perform a logical analysis, which is more or less a machine and then the logical results have to be transformed back to meaningful results in the empirical world. So we have two translations which so to say is the specific scientific job: translating concepts of reality into concepts of a theory/model and translating results of a logical/causal analysis of the theory/model into causality of the reality. On the road to the final logical/causal analysis we thus have different kinds of categorizations, conceptualizations, systematizations and similar matters but these matters are not scientific results per se but preliminaries to the causal logical analysis.

Figure 1: Illustration of Proposition 6.211



Important is to note that while the logical analysis per se is reversible the causal analysis is never so; it is strictly irreversible in Hume's sense. True that with respect to locally and temporally inert structures there may appear almost reversible structures on the microscopic levels, particularly in natural sciences as physics and chemistry. However when subjects are present in the analysis it is probably so that causality even at the microscopic level displays almost always irreversible features. This means, if we stick to the social sciences that any kind of dynamics is irreversible but this irreversibility is not present in the logical analysis per se but in the characterization of the concepts. Thus if someone says NO to another person's proposition but then changes it after some time to YES cannot probably assume that the relation to the other person would have been the same as if the answer had been YES from the very beginning. However if I move my cup of coffee to another place of my desk I would almost have a reversible process if I move it back, since the movements would probably not affect the environment *in a measurable way*. Should I on the other hand spill some coffee during the restructuring process we have obviously a strict irreversible process. Consequently the irreversibility of social sciences even on the microscopic level has much to do with the basic lack of stable measures due to the fact that subjects are to be treated as final causes, locally and temporally.

4. Reversibility and Aggregation in Economic Theory

We mentioned above that with respect to physical dynamic processes it would sometimes appear as if we have reversibility on the microscopic level but we reject it on macroscopic levels. The reason is of course that while some changes on the microscopic level appear as reversible due to almost un-measurable effects on the environment. On the macroscopic level it cannot be so since such a change would affect the microscopic structures and thus cause changes in the microscopic dynamics. In the fundamental economic theory, the neoclassical theory, which both has a microscopic level and a macroscopic we obviously have reversibility.

It can thus be of some value to study how this is achieved; obviously we must introduce some restrictions in the definition of the concepts. Let us therefore somewhat superficially go through the basic features of the neoclassical theory in the light of the above discussions. The neoclassical theory deals with the very principle of economic exchange.

It is often linked to Adam Smith's expression "the invisible hand" which alludes to the fact that the butcher does not sell a kilo of meat because of kindness but because he gets *something* in exchange, however what this *something* really is, is somewhat blurred. Money is natural to think of but during the times of Adam Smith it was only partially the case, often there was barter in some form, either in commodities but most often in form of corn since the remuneration of the workers mostly were paid in equivalences of corn. The money economy was developing and it is instructive to read David Hume's praise of the money economy, which to a large extent shows a great awareness of both the social role of money as well as the fact that the economic system was just a subsystem of the social reality and that it could not be separated from that. Thus the links between the neoclassical theory and Adam Smith are rather weak and the basic point is that the relative price of corn was tried to be kept constant though it was the basic measure of wealth in land. Furthermore Adam Smith kept the Aristotelian link of relative value of commodities as a mirror of relative imputed working hours. However when looking at the great economists of the end of the18th century and the beginning of the19th it is perfectly clear that they analyzed a *money economy*.

We may go back to the Dominican friars of Salamanca who were indeed puzzled by the role of money and so were David Hume, Jean Baptise Say, David Ricardo and Henry Thornton. The latter was actually far ahead of his time in analyzing the financial sector in his "Paper Credits in Britain" (1803). Ricardo kept the link between money and labor valuation while the others to a large extent, in the spirit of the friars of Salamanca, made the relative prices dependent of the actual context of the market exchange. This difference is indeed important because if we keep imputed labor per time unit as a fundamental anchor and fix the reimbursement of corn to land owners the relative prices will vary due to variations in the labour salary in corn equivalents but the variations, given that they are due to exogenous factors which are randomly distributed over time, will be limited and the economy will have a cyclical movement around a basic equilibrium like in the case of a Lotka - Volterra model (predator/prey), or perhaps even better illustrated by a
Phase-diagram of a Hopf - bifurcation in the two dimensional plane, **Figure 2**.

Figure 2: Hopf-bifurcation

In the world of the Friars of Salamanca, David Hume and Henry Thornton no such dynamics is available since the economic exchange in the money economy is only ruled by the local and temporal contexts of the agents and thus no general equilibrium exists. Jean Baptiste Say, who many economists wrongly regard as a neoclassical equilibrium economist has a most elegant rejection of a general equilibrium due to his rejection of the money price as a relevant measure:

When I am told that the great pyramid of Ghaize is 656 feet square at the base, I can measure a space of 656 feet square at Paris or elsewhere, and form an exact notion of the space the pyramid will cover; but when I am told that a camel is at Cairo worth 50 sequins, that is to say, about 90 ounces of silver, or 100 dollars in coin, I can form no precise notion of the value of the camel; because, although I may have every reason to believe that 100 dollars are worth less at Paris than at Cairo, I cannot tell what may be the difference of value.

Say (1834[1803]: 247)

Thus prices are a matter of the very context of the economic exchange. However this context might be more or less inert due to social and political circumstances. In eras of high social and political stability we might experience very stable price structures. Keynes has a very enlightening discussion on this matter in General Theory chapter 17 and 18 on the stability of wages and prices, but such stability depends on the social environment and social inertia and is not intrinsic to the exchange process per se.

In fact Keynes discussion is a sense closer to Adam Smith's discussion of price stability than the neoclassical analysis is, particularly in its axiomatic form. Adam Smith held the remuneration in corn equivalents to the land owner as more or less constant and the basic measure of relative prices was the imputed work hours. However although the salary to the worker per hour, month or so could vary due to exogenous factors the basic capacity of the workers was the primary production factor. Thus the deepest form of stabilizing factor was due to the survival of the workers on an economically efficient level. In this aspect Smith's analysis is also closer to Marx' analysis than to the neoclassical one.

5. Equilibrium – Disequilibrium

We have derived two development lines of economic thinking: the equilibrium and the disequilibrium approaches. Between these two modes of thinking there exists no possibility of compromise, they are contradictive in terms. Historically the clear division we see today when the neoclassical theory has obtained an axiomatic form is of a relatively late date. Although particularly Ricardo and even Smith were eager to follow Newton in what was believed the ultimate form of a scientific analysis, the concept of equilibrium is relatively blurred and the dependency of socio-political as well as cultural factors is partly explicitly and partly implicitly recognized. The turn to a mathematical treatment of equilibrium became more or less completed when the utility theory entered in a systematic form. It is very interesting to compare Mill's and Jevons' writings leading to Jevons 'mathematical formulation of a utility function for the individual.

Mill had ideas which strived towards a complete macroscopic formulation but had difficulties to get around the interrelations between individuals; his discussion actually points toward Arrow's paradox in some aspects (Mill 1990[1863]). Jevons realized these difficulties and defined utilities as well as individuals in atomistic terms and thus directly dismissed the parts which troubled Mill and could consequently find a way of representing the relation between the outer market supply and the interior of the individual given the rational individual. Jevons' achievements were still at the microscopic level but he obtained a consistent mathematical representation of the individual within certain limits. However at the macroscopic level Walras' counting of equations and commodities was the only way to achieve a macroscopic equilibrium and furthermore Pareto claimed that the utility theory worked within the greater forces which were of a sociological character. An eventual economic equilibrium was of an ephemeral character. The result was a rift between the mathematical representation of the macroscopic vis-à-vis the microscopic economics. The fully logically consistent representation of an economy populated with "rational" individuals was shaped during the period 1930s to 60s and reached an axiomatic form in Arrow& Debreu (1954).This was followed up by Debreu (1959) and Arrow & Hahn (1971) to mention some important texts and broadly speaking we might say that it reached its full axiomatic form in Debreu (1982) where the equivalence between Nash-equilibrium and Arrow-Debreu competitive equilibrium was shown.

It is worthwhile to notice that the theory often is used inter-temporally and this actually seems in line with Jevons although he does not say this explicitly. To the author it seems that the only ranking economist who *rejects* this is actually Gerard Debreu; we will come back to this later. So theoretically it took some 100 years to overcome the intrinsic rift in the neoclassical theory between the macroscopic world and the microscopic world and have a consistent approach dealing with the society *in toto*. Thus the microscopic entities could be consistently aggregated into a macroscopic representation of the economy. During this time however practical economists assumed that the rift was overcome earlier and freely applied the assumption of the rationality of the individual to macroscopic aspects. The most articulated is the rational expectation approach from the late 1960s.The disequilibrium approach is more difficult to get hold of. It contains Keynesianism, institutionalism, evolutionary economics and several other approaches that often build on analogies to other sciences as neurology, biology, and may be others. This is natural since contrary to equilibrium theory there are no unifying models for economic affairs only local and temporal, social, political and cultural forces.The equilibrium model has a most sophisticated mathematical form and thus seems to be the theory par preference.

The neoclassical general equilibrium means on one hand that the price vector for the defined commodity space is unique, given the distribution of initial distribution of commodities and/or productive resources. This means that the agents maximize their utilities/welfare given initial *endowments*. In early days these were regarded as a stock of commodities of different kind but in Makarov & Rubinov (1977) we got the final link between the Neumann-Gale production analysis and the Arrow/Debreu equilibrium thus we could replace (within the taken presumptions) commodities with productive resources and we could even differentiate between labour in qualitative terms.

One could believe that this made the model more flexible and realistic but either the productive resources must be exactly assigned to respective commodity or the productive resources must be perfectly homogenous. In the first case we might thus think of different equilibria dependent on exogenous factors and in the latter case the exogenous factors only affect volumes but not the relative prices. Consequently the neoclassical equilibrium theory has been developed to cover not only individual optimizing but a general equilibrium which also achieves the highest form of distributive efficiency, *Pareto efficiency*, such that if we are dissatisfied with it we need to shift the initial distribution of resources. The market process is subsequently neutral with respect to distribution. It is obvious that such an approach seems tasteful both from a scientific as well as an ideological/ethical point of view. With respect to ethics we implicitly can identify two opposite states; both build on the facts that the individuals act as independent atoms and that in equilibrium unanimously must rule.

That means that either we have a state of complete in-deterministic chaos or we have a strict rule for the individual to make all other individuals' preference functions restrictions for the own behavior which is rather similar to the Kantian imperative, *'Act externally in such a manner that the free exercise of thy will may be able to coexist with the freedom of all others, according to a universal law'* (Kant 2007[1795]:398) and as we see the Kantian imperative is quite consistent with a Nash-equilibrium, save for the dimensionality problem, which we will discuss later.

6. Axiomatic analysis

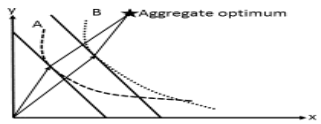
Let us however look at the general equilibrium theory in its axiomatic form. We will in this paper just make some comments and we assume that the reader is rather familiar with axiomatic treatment. We will also follow rather closely the scheme of the proof in (Ekstedt & Fusari: 57) in our exposé.

Table 1: The Neoclassical Axioms of Arrow/Debreu

Economic Rationality	**Properties of the Preference Ordering**
Axiom of Completeness *All commodities in the commodity space are known to everybody*	4. Axiom of Continuity
Axiom of Reflexivity *All commodities are identical to themselves*	5. Axiom of Convexity
Axiom of Transitivity *For any three commodities holds that if* $xRy \wedge yRz \Rightarrow xRz$	6. Axiom of Local Non-satiation *for any commodity basket A there exists at least one commodity* x_i *such that* $A(x_1,---, x_i+\varepsilon, ---, x_n) \succ A(x_1,---, x_i, ---, x_n)$

The neoclassical axiomatic structure as in **Table 1**has two parts: defining rationality and defining the properties of the preference ordering. There are three axioms in each group:The 6th axiom replaces the obsolete marginal decreasing utility hypothesis. From a mathematical point of view one might raise an (or even both) eyebrow because the axioms 1 to 5 actually define a Euclidian space. True that axiom 6 imputes something different, namely that the Euclidian space is directed. The implication is that the commodity space is seen as a Euclidian space where the **c**ommodities are the dimensions respectively and consequently the agents are represented by n-dimensional vectors. This implies that neoclassical aggregation is actually simple vector aggregation. Given a certain moment we may assume that all preferences are fixed. If we then impute a certain price vector over the commodity space we will have an optimum for all the agents with regard to the price vector. All agents will then optimize their budgets and we will have *m* vectors, which in the two-dimensional case can be illustrated as in **Figure 3**.

Figure 3: Additive vector aggregation in the plane



As a momentary picture this is no problem. All contextual considerations are done and we have to make the temporarily choice. That is actually what Gerard Debreu advocates (Debreu 1987[1959]: 29-32) when he claims that commodities must be indexed according to time and space. Expressed in such a way the general equilibrium approach is a quite acceptable description of the market process. But then the optimizing choice by the agents is not really a choice but an equilibration of choice criteria created in the contextual considerations and then effectuated when the ruling price vector is applied. Thus in the market exchange there is no real choice but a deterministic application of an optimizing rule. One might think of a joint creation of the price vector combination and true decision making and that is if we think of a flea-market limited in space and time and where the commodity space is given during the negotiation process, where the agents who have brought the junk negotiate mutually until a common price vector is found, a sort of Walrasian tâtonnement process. But as we see the time is lacking since we assume that the agents and the commodity space and the context is given. If we were to look at general equilibrium in this way we would have no possibility to explain a time trajectory. The evolution between different states of equilibrium would be almost random, probably with short intervals of *almost* equilibrium.

When we consider time we are in great trouble. Obviously we may assume that the context is given, at least in the meaning of a known space of outcome and a known probability distribution. Furthermore we may assume constant preference orders and commodity space. The point is that such an analysis feels a bit meaningless; it is hard to take it seriously, at least in the current state of the world, some of course we might have pockets of high inertia on the microscopic level but these are due to be swallowed by the macroscopic development. Still we may claim that we might use the neoclassical general equilibrium model as a kind of general but simplified model of the real economy and thus analytical derivations may give us hints of the direction in how to act with respect to the real economy in policy matters. Such a claim is the general claim of defense for the proponents of the neoclassical theory. To discuss this sort of claim we need to go deeper into the axiomatic structure and we must complete that with some comments on Arrow's paradox.

In relation to table 1 above, we pointed out that the axiomatic structure describes an ordered Euclidian space. Since that is what we normally work with in elementary mathematics it might seem very appropriate, but it is not. What we actually do is that we claim that the interior processes of the individual can be described in exactly the same way as the outer commodity space. Given Brower's dimension invariance theorem: P^n *is Homeomorphism to* P^m *if and only if* $n = m$ we may perform the same manipulation on the two spaces. Thus we lock the interior choice space of the individual point wise to the exterior commodity space. This was actually what Jevons' tried to do but the Arrow/Debreu approach is more efficient from a mathematical point of view. Applying Brower's theorem we however run into a terrible jumble when we try to introduce the time aspect. How do we actually handle the case where the dimensionality of the commodity space changes?

Keynes mentioned actually this problem in the preface to the French edition of General theory (Keynes 1973[1936]: xxxi-xxxv). This means that any change of the dimensionality of the commodity space will create a discontinuity, which of course can be assumed away in different ways but if so we enter into a social analysis outside the realm of mathematical considerations. So let us see how this is done in the axiomatic structure. The axiomatic structure ends in two important principles: the independence of irrelevant alternatives and the principle of revealed preferences. Both these principles grant the reversibility of choices and actions. Thus if I at one moment of time choose an alternative this will have no impact of my future choices.

Anyway I want to emphasize one particular axiom as very problematic and which determines the character of the whole analysis: axiom 2 concerning reflexivity. That means that a commodity is identical to itself. This axiom has often been regarded as trivial. Hausman, D.M. (2012: footnote 1 p. 13) writes:

Reflexivity is trivial and arguably a consequence of completeness, whereas continuity, which is automatically satisfied for any finite set of alternatives, is needed to prove that preferences can be represented by a continuous utility function.

There are two problematic statements in this quote; the first is of course the triviality and the second is that reflexivity follows from the axiom of completeness. To start with the latter one it is simply not true from a mathematical point of view. In an axiomatic structure every single axiom brings a particular aspect which is necessary to obtain the particular structure. Anyway the reflexivity axiom states that a certain commodity is identical to itself, which seems plausible.

The point here is however that we deal with two different spaces; the interior choice space of the agent and the exterior commodity space and we try to define the first one so that we can use the Euclidian space for both, given Brower's dimensionality theorem but then the things become a bit problematic. With respect to the outer commodity space we only need to deal with the physical characteristics of a commodity, which also applies to the production sector of the economy; the firms only produce physical things/processes. From the agents point of view it is however different. The commodities are seen in relation to the specific context which means that binary opposite rankings may occur given the same commodity at a different space-time point. This is actually why Debreu, who is basically a mathematician made his specification of commodities in space and time, which of course seems correct vis-à-vis the real world. This is essentially a fundamental problem for the economic analysis. Given this rift between the demand and the supply side we have to expect/accept non-unique price vectors. Furthermore it invalids any form of additive aggregation of agents and/or commodities.

7. Arrow's paradox

This brings us to consider Arrow's paradox. Unfortunately this paradox has been interpreted with respect to the real world which is somewhat peculiar. All logical paradoxes concern a precise axiomatic structure and a case where concepts are brought to exhaust the structure some way or another. Thus the axioms assumed by Arrow are a sort of social generalization of the individual axioms, see Ekstedt (2015[2012]:70-74). As we now see the axiom of reflexivity affects the analysis by making the contextual matters insignificant and the only thing that matters is the price vector since the individuals are actually locked into their preference structure and *do not actually* choose since that is done by the price vector (if it is the invisible hand or somebody else controlling it is outside our scope here). Thus given a preference structure (explicitly or implicitly assumed) the optimization process is purely deterministic. Consequently Arrow populates the economy with agents who are not able to choose and then we have the ordinary general equilibrium with the Pareto efficiency criterion fulfilled. Then he imputes a new kind of agents who are able to negotiate and make compromises, thus they are context dependent. By doing so he shows that the neoclassical general equilibrium does not hold when these latter agents are present in the analysis.

8. Final conclusions

Much has been written about Arrow's paradox in relation to the real world and some of it is surely valuable and good but the basic matter is that if we impute agents who have some resemblance with everyday people and are context dependent than the axiom of reflexivity is broken and a general equilibrium built on the neoclassical axiomatic structure in its Arrow/Debreu form cannot exist. Thus the way of aggregating individual market actions into behavioral patterns of the economy as a whole which is a part of the social structure does not hold. The reasons are basically the representation of individual agents as some kind of rational atoms which irrespective of contextual matters make the market choices. Obviously Debreu's approach that the so called rational market actions occur after the individual has made their contextual considerations, leads to the conclusion that rationality has two aspects.

First it contains what we may call the animal rationality which might be well described by the axioms 1 through 3 in table one, but secondly the individual has to make contextual considerations to make a base for the animal rationality. If we consider the two aspects we may change the word *rationality* to *purposeful action* but then we also unleash the concept from the mathematical concept defined by the axioms. Since logics/mathematics is empty of any content in the real world the contextual matters will make rationality just temporary and local and nothing prevents that we have two consecutive actions contradicting each other since the context is changed. This will also have some philosophical bearing on the difference between Kant and Hume where Kant claims reason as the true human ability and the ability to consider the true reality apart from ephemeral emotional sensations while Hume claims that reason is the humble servant of passion. Thus the governing forces of humans according to Hume are the will, the passions, the ideas, all for good and evil. This has a peculiar effect, namely that since we cannot judge the contextual apprehensions by the individuals we cannot per se regard any action as irrational in the neoclassical sense. Consequently a contextual apprehension built on ignorance and prejudice may still be rational. Technically speaking the neoclassical axiomatic structure defines a commodity space which is strictly positive. That means that all commodities are strictly positive to all agents. This is actually a consequence of the axiom of reflexivity which many economists see as trivial. In the real world, as we said the axiom of reflexivity holds when commodities are regarded just as physical items but as commodities they have to be used as means in a particular context.

Jevons and Mill understood this, but nowadays this understanding has been blurred by pure technical/mathematical manipulations. As some form of conclusion we may say that although the neoclassical optimization approach may well describe the individual behavior any form of aggregation to the economy as a whole is basically built on false logics.

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